3.5 Rational Functions

❖ Rational Functions

<u>Rational Functions</u>: quotients of polynomial functions

$$f(x) = \frac{p(x)}{q(x)},$$

where p and q are polynomial functions and $q(x) \neq 0$.

<u>Domain</u> of a Rational Function: the set of all real numbers <u>except the *x*-values that make the denominator zero</u>.

To Find the Domain of a Rational Function: (Sec. 2.8)

1) Set the **denominator = 0** and **solve for *x***.
2) **<u>Exclude</u>** the resulting real values of **<u>*x* from the domain</u>**.

Ex. Find the domain of each rational function. Write the domain in interval notation.

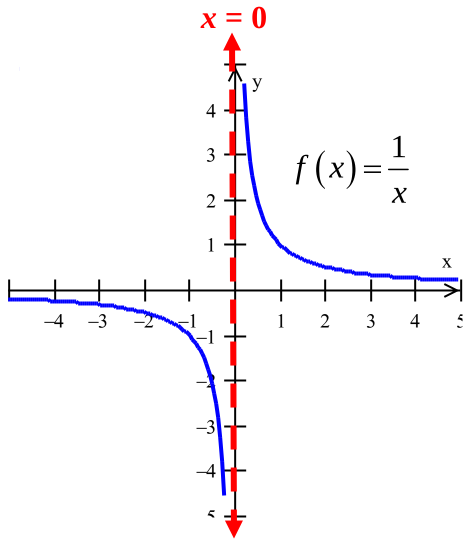
(a) $f(x) = \dfrac{-3x+7}{5x-2}$

(b) $F(x) = \dfrac{x+2}{x^2+4}$

(c) $R(x) = \dfrac{x^2}{x^2+x-6}$

(d) $H(x) = \dfrac{x^2-1}{x+1}$

❖ Vertical Asymptotes of Rational Functions



<u>Vertical Asymptotes</u>: The line $x = a$ is a vertical asymptote of the graph of a function *f* if $f(x)$ increases or decreases without bound as *x* approaches *a*.
Thus, as *x* approaches *a* from either the left or the right.
(There can be more than one vertical asymptote or none at all. A graph can never intersect a vertical asymptote.)

To Find the Vertical Asymptotes $(x = a)$**:**

1) Simplify the rational function to its **lowest term**.

   If $f(x)=\frac{p(x)}{q(x)}$ is a rational function and if *p* and *q* have <u>no common factors</u>, then the rational function *f* is said to be in **<u>lowest term</u>**.
2) Set the **denominator = 0** and solve for *x*.

Ex. Find the vertical asymptotes, if any, of the graph of each rational function.

(a) $f(x)=\frac{-3x+7}{5x-2}$

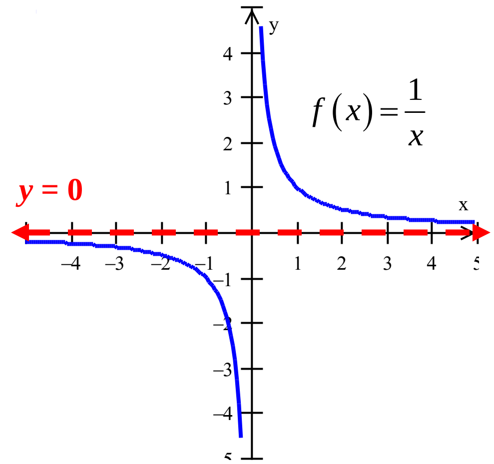
(b) $F(x)=\frac{x+2}{x^2+4}$

(c) $R(x)=\frac{x^2}{x^2+x-6}$

(d) $H(x)=\frac{x^2-1}{x+1}$

❖ Horizontal Asymptotes of Rational Functions

<u>Horizontal Asymptote</u>: The line $y = b$ is a horizontal asymptote of the graph of a function *f* if $f(x)$ approaches *b* as *x* increases or decreases without bound. (It can have <u>at most one horizontal asymptote</u> or <u>none at all</u>. A graph may cross its horizontal asymptote.)



To Find the Horizontal Asymptotes $(y = b)$:

Compare the highest degree in the numerator (***n***) to the highest degree in the denominator (***m***).

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1.) $n < m$: The horizontal asymptote is $y = 0$ (the *x*-axis).

2.) $n = m$: The horizontal asymptote is $y = \frac{a}{b}$ (***a***: the leading coefficient of the numerator; ***b***: the leading coefficient of the denominator).

3.) $n > m$: There is <u>no horizontal asymptote</u>.

Ex. **i)** Find the horizontal asymptote, if any, of the graph of each rational function.
ii) If the graph of the function has a horizontal asymptote, determine the point where the graph crosses the horizontal asymptote.

(a) $f(x) = \frac{-3x+7}{5x-2}$

(b) $F(x) = \frac{x+2}{x^2+4}$

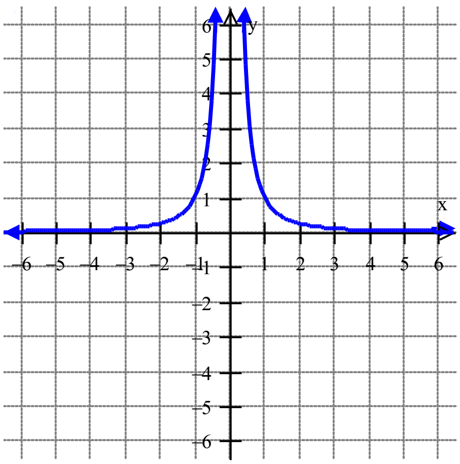
(c) $R(x) = \frac{x^2}{x^2+x-6}$

(d) $H(x) = \frac{x^2-1}{x+1}$

❖ Using Transformations to Graph Rational Functions

Ex. Use the graph of $f(x)=\frac{1}{x^2}$ to graph $T(x)=\frac{2}{(x+2)^2}+1$. State the vertical asymptote(s) and horizontal asymptote of each function.

$f(x)=\frac{1}{x^2}$



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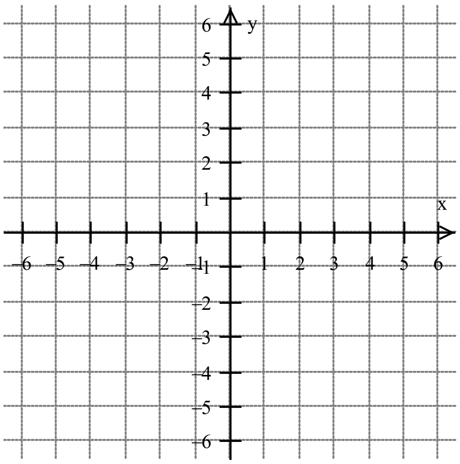
$T(x)=\frac{2}{(x+2)^2}+1$



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❖ Slant (Oblique) Asymptotes of Rational Functions

Slant (Oblique) Asymptote: If the degree of the numerator is **one** more than the degree of the denominator (***n*** **=** ***m*** **+ 1**), then the graph has a slant asymptote, $y=mx+b$. The equation of the slant asymptote can be found by ***division***.

$$f(x)=\frac{p(x)}{q(x)}=mx+b+\frac{\text{remainder}}{q(x)}$$

Ex. Find the slant asymptote of the graph of each rational function.

(a) $f(x)=\frac{x^2-4}{x}$

(b) $f(x)=\frac{-2x^2-3x+7}{x+3}$

(c) $f(x)=\dfrac{4x^3-2x^2+7x-3}{2x^2+4x+3}$